Differentiability

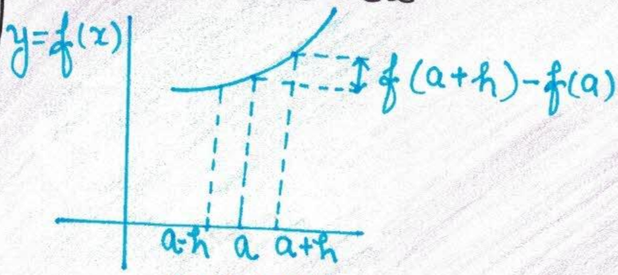
Differentiability

Derivative of a function $y = f(x)$ is defined as rate of change in $f(x)$ w.r.t an independent variable x.

In $y = f(x)$

y → dependent variable

x → independent variable



$$\frac{\Delta y}{\Delta x} = \frac{f(a+h)-f(a)}{h}$$

$\Delta x = a+h-a$

$= h$

Right hand derivative :→

$$Rf'(x)\Big|_{x=a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

Left hand derivative :→

$$Lf'(x)\Big|_{x=a} = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$$

★ A function $f(x)$ is Said to be differentiable at $x = a$ if its LHD = RHD

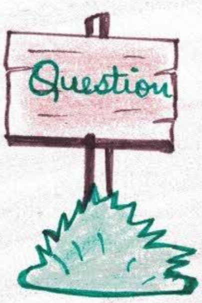
★ If a function is differentiable at $x = a$ then it must be Continuous at $x = a$.

But the converse is not true.

$$f'(a) = \lim_{x \to 0} \frac{f(a+x) - f(a)}{x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

This is derivative of $y = f(x)$ by First principle



$$y = \sin x$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(x + \frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\cos(x + h/2)\sin h/2}{h/2}$$

$$= \cos(x + h/2)$$

$$= \cos x$$

Question

$$y = a^x$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\left(a^{x+h} - a^x\right)}{h}$$

$$= \lim_{h\to 0} \frac{a^x\left(\frac{a^{x+h}}{a^x} - 1\right)}{h} \Rightarrow \lim_{h\to 0} \frac{a^x(a^h - 1)}{h}$$

$$\Rightarrow \boxed{a^x \ln a}$$

Question

$$y = \log x$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\log(x+h) - \log x}{h}$$

$$= \lim_{h\to 0} \frac{\log\left(\frac{x+h}{x}\right)}{h} = \lim_{h\to 0} \frac{\log\left(1+\frac{h}{x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\log\left(1+\frac{h}{x}\right)}{\frac{h}{x}\times x} \Rightarrow \frac{1}{x}$$

Question

$$\tan(A+B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$f(x) = \tan x$$

$$\frac{dy}{dx} = \frac{\tan(x+h) - \tan x}{h}$$

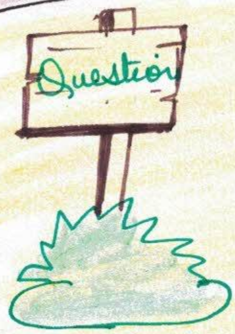
$$= \frac{\tan(x+h-x)\left[1+\tan(x+h)\tan x\right]}{h}$$

$$= \frac{\tan h\left[1+\tan(x+h)\tan x\right]}{h}$$

$$= 1+\tan(x+h)\tan x$$

$$= 1+\tan^2 x$$

$$= \sec^2 x$$



$$f(x) = \sin \log (x)$$

$$\frac{dy}{dx} = \frac{\sin \log (x+h) - \sin \log x}{h}$$

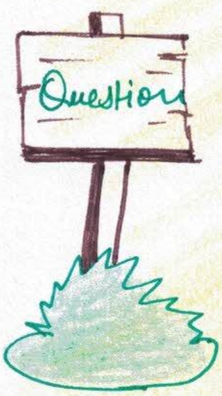
$$= \frac{2 \cos \left[\frac{\log (x+h) + \log x}{2}\right] \sin \left[\frac{\log (x+h) - \log x}{2}\right]}{h}$$

$$= \frac{2 \cos \left[\frac{\log x^2}{2}\right] \sin \left[\frac{\log \left(1+\frac{h}{2}\right)}{2}\right]}{h}$$

$$= \frac{2}{h} \cos [\log x] \sin \left[\frac{\log \left(1+\frac{h}{x}\right)}{\frac{h}{x} \times \frac{x}{h} \times 2}\right]$$

$$= \frac{2 \cos (\log x) \sin \left(\frac{h}{2x}\right)}{h} \Rightarrow \frac{\frac{2 \cos \log x \sin \frac{h}{2x}}{h}}{\frac{h}{2x} \times \frac{2x}{h}}$$

$$= \frac{2 \cos \log x}{h} \times \frac{h}{2x} = \boxed{\frac{2 \cos (\log x)}{x}}$$



$$f(x) = x^m$$

$$\frac{dy}{dx} = \frac{(x+h)^n - x^n}{h} = \frac{x^n \left[\left(1+\frac{h}{x}\right)^n - 1\right]}{h}$$

$$= \frac{x^n}{h} \left[1 + \frac{nh}{x} + \frac{n(n-1)}{2!}\left(\frac{h}{x}\right)^2 + \dots\dots - 1\right]$$

$$= \frac{x^n}{h} \left[\frac{nh}{x} + \frac{n(n-1)}{2!} \frac{h^2}{x^2} \text{------}\right]$$

$$= \boxed{n x^{n-1}}$$

Question

$$\sqrt{\tan\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{\sqrt{\tan\sqrt{x+h}} - \sqrt{\tan\sqrt{x}}}{h}$$

$$= \frac{\sqrt{\tan\sqrt{x+h}} - \sqrt{\tan\sqrt{x}}}{h} \times \frac{\sqrt{\tan\sqrt{x+h}} + \sqrt{\tan\sqrt{x}}}{\sqrt{\tan\sqrt{x+h}} + \sqrt{\tan\sqrt{x}}}$$

$$= \frac{\tan\sqrt{x+h} - \tan\sqrt{x}}{h \times 2\sqrt{\tan\sqrt{x}}} \qquad \tan(A-B) = \frac{\sin(A-B)}{\cos A \cos B}$$

$$= \frac{\sin(\sqrt{x+h} - \sqrt{x})}{\cos\sqrt{x+h}\,\cos\sqrt{x} \times h \times 2\sqrt{\tan\sqrt{x}}}$$

$$\Rightarrow \frac{\sin(\sqrt{x+h} - \sqrt{x}) \times (\sqrt{x+h} - \sqrt{x})}{(\sqrt{x+h} - \sqrt{x}) \times \cos\sqrt{x+h}\,\cos\sqrt{x} \times h \times 2\sqrt{\tan\sqrt{x}}}$$

$$= \frac{\sqrt{x+h} - \sqrt{x}}{\cos\sqrt{x+h}\,\cos\sqrt{x} \times 2\sqrt{\tan\sqrt{x}} \times h} = \frac{\sqrt{x+h} - \sqrt{x}}{\cos^2\sqrt{x} \times 2\tan\sqrt{x} \times h}$$

$$= \frac{\sec^2\sqrt{x} \times (\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})\, 2\sqrt{\tan\sqrt{x}} \times h}$$

$$= \frac{\sec^2\sqrt{x} \times h}{(\sqrt{x+h} + \sqrt{x})\, 2(\tan\sqrt{x} \times h}$$

$$\Rightarrow \boxed{\frac{\sec^2\sqrt{x}}{4\sqrt{x}\sqrt{\tan\sqrt{x}}}}$$

Question

$f(x) = e^x$

$$\frac{dy}{dx} = \frac{e^{x+h} - e^x}{h} \Rightarrow \frac{e^x(e^h - 1)}{h}$$

$$\Rightarrow \boxed{e^x}$$

Question

$f(x) = \cos^{-1} x$

Let $x = \cos y$

$$\frac{dy}{dx} = \frac{\cos^{-1}(x+h) - \cos^{-1} x}{h}$$

$$= 2 \sin\left(\frac{2y+h}{2}\right) \sin\left(\frac{-h}{2}\right)$$

$$= -\sin\left(\frac{2y+h}{2}\right) \sin\frac{h}{2} = -\sin\left(\frac{2y+h}{2}\right)$$

$$= -\sin y \Rightarrow -\sqrt{1-x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}}$$

Question

$f(x) = \tan^{-1} x$

$1 + x^2 = \sec^2 y.$

Let $\tan y = x$

$$\frac{dx}{dy} = \frac{\tan(y+h) - \tan y}{h}$$

$$\Rightarrow \frac{\sin(y+h-y)}{\cos(y+h)\cos y\, h} = \frac{\sin h}{\cos(y+h)\cos y\, h}$$

$$= \frac{1}{\cos^2 y} = (\sec y)^2 = \left(\sqrt{1+x^2}\right)^2$$

$$= 1 + x^2$$

$$\boxed{\frac{dy}{dx} = \frac{1}{1+x^2}}$$

Question $f(x) = \sec x.$

$$y = \sec x$$

$$\frac{dy}{dx} = \frac{\sec(x+h) - \sec x}{h} = \frac{\cos x - \cos(x+h)}{h \cos(x+h) \cos x}$$

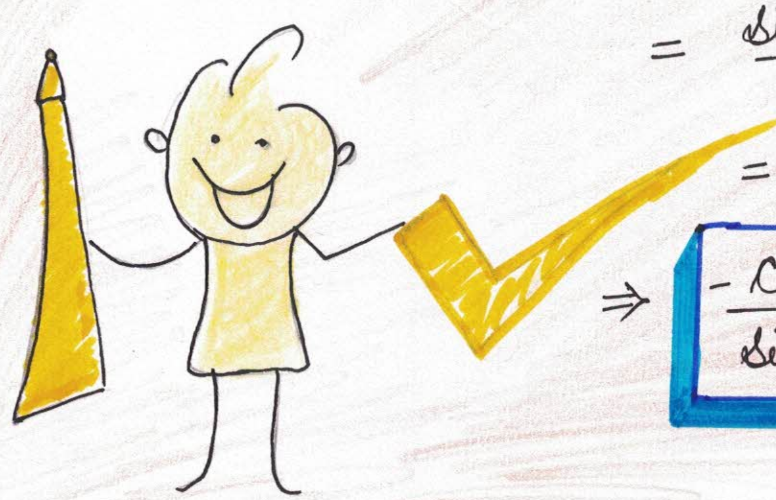
$$\Rightarrow \frac{\sin\left(\frac{2x+h}{2}\right) \sin\left(\frac{+h}{2}\right)}{\frac{h}{2} \cos(x+h) \cos x} = \frac{\sin x}{\cos^2 x}$$

$$\boxed{\frac{dy}{dx} = \tan x \sec x}$$

Question $f(x) = \operatorname{cosec} x$

$$\frac{dy}{dx} = \frac{\operatorname{cosec}(x+h) - \operatorname{cosec} x}{h}$$

$$= \frac{\sin x - \sin(x+h)}{\sin(x+h) \sin x \times h}$$



$$= -\cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{+h}{2}\right)$$

$$\Rightarrow \boxed{\frac{-\cos x}{\sin^2 x} = -\cot x - \operatorname{cosec} x}$$

Question

$f(x) = \sec^{-1}x$

$y = \sec^{-1}x \Rightarrow x = \sec y$ $y \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$$\frac{dx}{dy} = \frac{\sec(y+h) - \sec y}{h} = \frac{\cos y - \cos(y+h)}{\cos^2 y \times h}$$

$$\Rightarrow \frac{2 \sin y \sin \frac{h}{2}}{\cos^2 y\, h} \Rightarrow \frac{\sin y}{\cos^2 y}$$

$$= \tan y \sec y \Rightarrow |x|\sqrt{x^2-1}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}}$$

Question

$f(x) = \operatorname{cosec}^{-1}x$

$y \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\operatorname{cosec} y = x$

$$\frac{dx}{dy} = \frac{\operatorname{cosec}(y+h) - \operatorname{cosec} y}{h} = \frac{\sin y - \sin(y+h)}{\sin^2 y\, h}$$

$$\Rightarrow -2\cos\left(y+\frac{h}{2}\right)\sin\left(h/2\right) = -\frac{\cos y}{\sin^2 y}$$

$$\Rightarrow -\cot y \operatorname{cosec} y$$

$$= -\sqrt{x^2-1}\; x.$$

$$\boxed{\frac{dy}{dx} = \frac{\pm 1}{x\sqrt{x^2-1}} = \frac{1}{|x|\sqrt{x^2-1}}}$$

Question $f(x) = \cot^{-1}x$

$$y = \cot^{-1}x \Rightarrow x = \cot y$$

$$\frac{dx}{dy} = \frac{\cot(y+h) - \cot y}{h} = \frac{\tan y - \tan(y+h)}{\tan^2 y\, h}$$

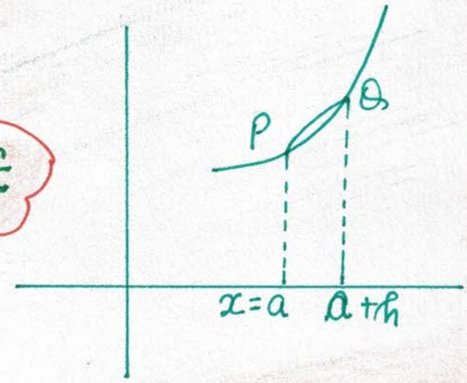
$$= \frac{\sin(-h)}{\cos^2 y \tan^2 y \times h} = \frac{1}{\sin^2 y} = -\operatorname{cosec}^2 y = -\left(\sqrt{1+x^2}\right)^2$$

$$\frac{dx}{dy} = -(1+x^2) \qquad \boxed{\frac{dy}{dx} = \frac{-1}{1+x^2}}$$

GEOMETRICAL MEANING OF DERIVATIVE OF A FUNCTION AT A POINT

$y = f(x)$

RIGHT HAND DERIVATIVE



$P = (a, f(a))$

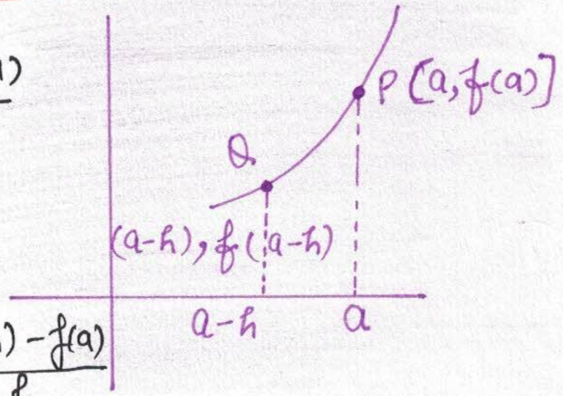
$Q = (a+h, f(a+h))$

Slope of Chord PQ $= \dfrac{f(a+h) - f(a)}{a+h-a} = \dfrac{f(a+h) - f(a)}{h}$

$$\lim_{h\to 0} (\text{Slope of Chord } PQ) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}.$$

$$\boxed{\text{Slope of tangent } PQ \text{ from right} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}}$$

LEFT HAND DERIVATIVE



$$\text{Slope of chord } PQ = \frac{f(a-h)-f(a)}{a-h-a}$$

$$= \frac{f(a-h) - f(a)}{-h}$$

$$\lim_{h\to 0} (\text{Slope of Chord } PQ) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$$

$$\lim_{h\to 0} \left(\text{tangent's slope at } P \text{ from left}\right) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$$

So, a function $y = f(x)$ is differentiable at $x = a$

if LHD = RHD

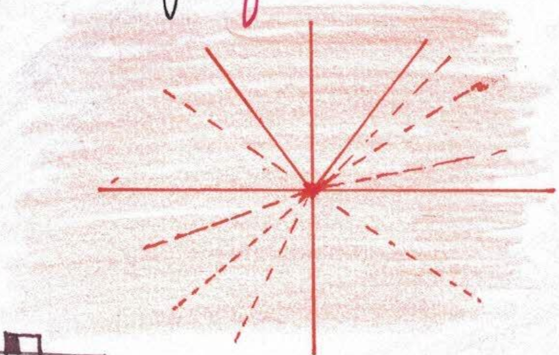
slope of tangent at $x = a$ from left = slope of tangent at $x=a$ from right

Hence, there exists a unique tangent at $x = a$.

Curve is smooth at $x = a$ where $f(x)$ is differentiable.

If $f(x)$ from any corner point at $x=a$, then $f(x)$ is not differentiable at that Point because we cannot draw a unique tangent at that point.

e.g $f(x) = |x|$ is not differentiable at $x=0$.

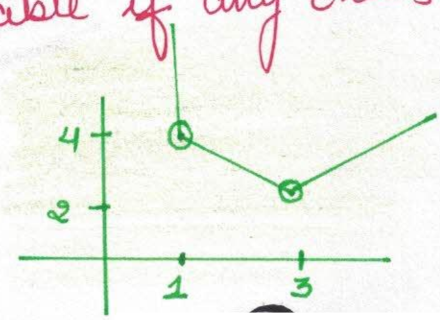


$\{\infty$ tangents can be drawn$\}$

Question

Find the pts. of non differentiable if any Exists.

(i) $f(x) = |x-1| + 2|x-3|$



$\rightarrow$ Non differentiable at 1 & 3.

(ii) $f(x) = |x-1| + |x-2| + |x-4|$

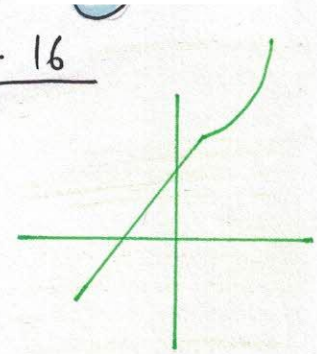
$\rightarrow$ Non differentiable at 1, 2, 4.

(iii) $f(x) = \begin{cases} x+4 & , x \le 4 \\ x^2-8 & , x > 4 \end{cases}$

$$LHD \Rightarrow \frac{(4-h+4)-8}{-h} = \frac{-h}{-h} = 1$$

$$RHD \Rightarrow \frac{(4+h)^2-8-(16-8)}{h} = \frac{(4+h)^2-16}{h}$$

$$\Rightarrow \frac{(4+h+4)(4+h-4)}{h} = 8$$



LHD $\neq$ RHD Non differentiable

DIFFERENTIATION OF A FUNCTION IN AN INTERVAL

Let $f(x)$ is defined in $[a, b]$

→ If $f(x)$ is differentiable at each point in $[a, b]$ then we say that $f(x)$ is differentiable in $[a, b]$.

→ If $f(x)$ is differentiable at each point in (a, b) and if $f'(a^+)$ & $f'(b^-)$ Exists then $f(x)$ is differentiable in $[a, b]$.

e.g. check the differentiability as well as continuity.

1. $f(x) = \begin{cases} x^2-1, & x \leq -1 \\ 2x+1, & x > -1 \end{cases}$ at $x = -1$

→ $-1^-,\ -1-h \Rightarrow (-1-h)^2 - 1 \Rightarrow +1-1 = 0$

$-1,\ 1-1 = 0$

$-1^+,\ -1+h \Rightarrow 2(-1+h)+1 \Rightarrow -2+1 = 1$

Not continuous, not differentiable.

2. P.T. $f(x) = |x^2-4|$ is not differentiable at $x=2$

→ $LHD \Rightarrow \dfrac{|(2-h)^2-4|}{-h} = \ominus$

$RHD \Rightarrow \dfrac{|(2+h)^2-4|}{h} = \oplus$

$\therefore\ LHD \neq RHD$

It is not differentiable

3. Check for cont. / diff.

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases} \quad \text{at } x = 0$$

$\rightarrow \quad 0^- \Rightarrow -h \Rightarrow h^2 \sin\left(-\frac{1}{h}\right) \Rightarrow -h^2 \sin\frac{1}{h}$

$$\Rightarrow \frac{-h^2 \sin\frac{1}{h}}{\frac{1}{h} \times h} = \underline{\underline{0}}$$

$0^+ \Rightarrow h \Rightarrow h^2 \sin\left(\frac{1}{h}\right) = \underline{\underline{0}}$

$x = 0 \Rightarrow y = 0$

Hence, continuous

$$LHD = -h^2 \sin\left(\frac{1}{h}\right) = h \sin\left(\frac{1}{h}\right)$$

$$RHD = \frac{h^2 \sin\left(\frac{1}{h}\right)}{h} = h \cdot \sin\left(\frac{1}{h}\right)$$

LHD = RHD

Differentiable at $\underline{\underline{x = 0}}$

Question

P. T. $f(x) = x|x|$ is differentiable at $x = 0$.

$$\rightarrow \quad LHD = \frac{f(-h) - f(0)}{-h} = \frac{-h^2}{-h} = h$$

$$\rightarrow \quad RHD = \frac{f(h) - f(0)}{h} = \frac{h^2}{h} = h$$

LHD = RHD, Hence differentiable.

Question

Check for continuity / Differentiability of

$$f(x) = \begin{cases} x \sin(\log x^2), & x \neq 0 \\ 0, & x = 0 \end{cases} \quad \text{at } x = 0$$

$\rightarrow$ 0^-, $-h \sin \log h^2 \rightarrow 0$

0^+, $h \sin \log h^2 \rightarrow 0$

At 0, $f(x) = 0$

Hence continuous

$$\text{LHD} \Rightarrow \frac{-h \sin(\log h^2) - 0}{-h} = \sin \log(h^2)$$

$$\text{RHD} \Rightarrow \frac{h \sin \log(h^2) - 0}{h} = \sin \log(h^2)$$

Hence differentiable

Question

If $f(x) = \begin{cases} ae^x + b - \cos x, & x \leq 0 \\ bx + 2a, & x > 0 \end{cases}$

is diff. at $x = 0$. Find $a + b$.

$\rightarrow$ Applying continuity at $x = 0$,

$$a + b - 1 = 2a$$

$$b - 1 = a \quad \text{------ (1)}$$

function is differentiable,

$$f'(x) = \begin{cases} ae^x + \sin x \\ b \end{cases}$$

At $x = 0$, $\boxed{a = b}$ .... (2)

$$b - 1 = a$$

$$a - 1 = a \qquad \boxed{0 \neq -1}$$

Hence, no value of a, b exists for such case.

▶ $Sgn(x)$ is discontinuous at $x=0$. Thus, $Sgn(f(x))$ is discontinuous at $x=a$, $\underline{f(a)=0}$

DIFFERENTIATON OF FUNCTION

CHAIN RULE

$y = f(u)$ where u is a function of x.

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

e.g $y = f(x^2+2x-3)$

$u = x^2-2x-3.$

$\frac{du}{dx} = 2x+2$

$\frac{du}{dx} = f'(x^2+2x-3)$

$\rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^2+2x-3)$

$= 2x+2$

$$\boxed{= 2x+1)\, f'(x^2+2x-3}$$

Question

$y = f(\sin^2 x)$ & $f'(x) = \frac{1+x}{1-x}$ then find $\frac{dy}{dx}$.

$u = \sin^2 x$

$\frac{du}{dx} = 2\sin x \cos x$

$\frac{dy}{dx} = f'(u) \times 2\sin x \cos x \Rightarrow \frac{1+\sin^2 x}{1-\sin^2 x} \times 2\sin x \cos x$

$$\boxed{= 2(1+\sin^2 x)\tan x}$$

If $y = f(t)$ & $x = g(t)$.

then $\frac{dy}{dx} = f'(t)$, $\frac{dx}{dt} = g'(t)$

$$\frac{dy}{dx} = \frac{f'(t)}{g'(t)}$$

Question

$y(t) = t^2 + 2at$, $x(t) = \sin t + 2t^2$. find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 2t + 2a$$

$$\frac{dx}{dy} = \cos t + 4t$$

$$\frac{dy}{dx} = \frac{2(t+a)}{\cos t + 4t}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{dx/dt}$$

If $t = y \Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}}$

PRODUCT RULE ....

$$y = f(x)\, g(x)$$

$$\boxed{\frac{dy}{dx} = \frac{g(x) f'(x) - f(x) g'(x)}{[g(x)]^2}}$$

DIFFERENTIATION OF LOGARITHMIC FUNCTION

Suppose $f(x) = \dfrac{A_1(x)\,A_2(x)}{A_3(x)\,A_4(x)}$ where A_1, A_2, A_3, A_4 are all differentiable functions in x & $A_1(x), A_2(x), A_3(x)$ & $A_4(x) \neq 0$,

then $\dfrac{u}{v}$ method is hectic.

So, we take log on both sides,

$$\log f(x) = \log A_1 + \log A_2 - \log A_3 - \log A_4$$

Now differentiating

$$\frac{f'(x)}{f(x)} = \frac{A_1'(x)}{A_1(x)} + \frac{A_2'(x)}{A_2(x)} - \frac{A_3'(x)}{A_3(x)} - \frac{A_4'(x)}{A_4(x)}$$

$$\boxed{f'(x) = f(x)\left[\frac{A_1'(x)}{A_1(x)} + \frac{A_2'(x)}{A_2(x)} - \frac{A_3'(x)}{A_3(x)} - \frac{A_4'(x)}{A_4(x)}\right]}$$

Question

Find $\dfrac{dy}{dx}$ if $y = \dfrac{(2x-3)^2\sqrt{3x^2-1}}{(x^2+4)\,2^x}$

$$\longrightarrow \log y = 2\log(2x-3) + \frac{1}{2}\log(3x^2-1) - \log(x^2+4)\ 3\log 2$$

$$\frac{1}{y}\frac{dy}{dx} = 2\times\frac{1\times 2}{2x-3} + \frac{1\times 6x}{2(3x^2-1)} - \frac{2x}{x^2+4} - \log 2$$

$$\boxed{\frac{dy}{dx} = \frac{(2x-3)^2\sqrt{3x^2-1}}{(x^2+4)\,2^x}\left[\frac{4}{2x-3} + \frac{3x}{3x^2-1} - \frac{2x}{x^2+4} - \log 2\right]}$$

Question $y = \left(x+\frac{1}{x}\right)^{x} + (x)^{\left(1+\frac{1}{x}\right)}$

$$\log y = x\log\left(x+\frac{1}{x}\right) + \left(1+\frac{1}{x}\right)\log x.$$

$$\frac{1}{y}\frac{dy}{dx} = \left\{x \times \frac{1 \times \left(1-\frac{1}{x^2}\right)}{x+\frac{1}{x}} + \log\left(x+\frac{1}{x}\right)\right\} + \left\{\left(1+\frac{1}{x}\right)\frac{1}{x} + \log x\left(-\frac{1}{x^2}\right)\right\}$$

$$\frac{1}{y}\frac{dy}{dx} = \left\{\frac{x}{x+\frac{1}{x}}\left(1-\frac{1}{x^2}\right) + \log\left(x+\frac{1}{x}\right)\right\} + \left\{\left(\frac{1}{x}+\frac{1}{x^2}\right) - \log(x)\frac{1}{x^2}\right\}$$

$$\boxed{\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^{x} + (x)^{1+\frac{1}{x}}\left\{\frac{x}{1+\frac{1}{x}}\left(1-\frac{1}{x^2}\right) + \log\left(1+\frac{1}{x}\right) + \left(\frac{1}{x}+\frac{1}{x^2}\right) - \frac{1}{x^2}\log x\right\}}$$

Question $y = a^{x^{a^{x^{\cdots\infty}}}}$

$$y = a^{x^{y}}.$$

$$\log y = x^{y}\log a$$

$$\log(\log y) = \log(x^{y}\log a)$$

$$\log(\log y) = y\log x + \log(\log a)$$

$$\frac{1}{\log y} \times \frac{1}{y}\frac{dy}{dx} = \frac{y}{x} + \log x\frac{dy}{dx} + 0$$

$$\frac{dy}{dx}\left(\frac{1}{y\log y} - \log x\right) = \frac{y}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{y^2\log y}{x(1 - y\log x\log y)}}$$

Question

$$y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$$

$$y = \tan^{-1}\left(\frac{a - b\tan x}{b + a\tan x}\right) \Rightarrow \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \left(\frac{a}{b}\right)\tan x}\right)$$

$$y = \tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}(\tan x)$$

$$y = \tan^{-1}\left(\frac{a}{b}\right) - x \Rightarrow \boxed{\frac{dy}{dx} = -1}$$

$$\boxed{y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)}$$

$$x = \sin\theta$$

$$\frac{x}{1+\sqrt{1-x^2}} = \frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta}{2\cos^2\theta/2} = \tan\frac{\theta}{2}$$

$$\tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{\sin^{-1}(x)}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{d(\sin^{-1}x)}{dx} \Rightarrow \boxed{\frac{1}{2\sqrt{1-x^2}}}$$

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$$

$$x = \tan\theta$$

$$y = \sin^{-1}(\sin 2\theta) + \sec^{-1}(\sec 2\theta)$$

$$= 4\theta = 4\tan^{-1}x$$

$$\boxed{\frac{dy}{dx} = \frac{4}{1+x^2}}$$

Question

$$y = \tan^{-1}\left(\frac{4x}{1+5x^2}\right) + \tan^{-1}\left(\frac{2+3x}{3-2x}\right)$$

$$y = \tan^{-1}\left(\frac{5x-x}{1+5x^2}\right) + \tan^{-1}\left(\frac{\frac{2}{3}+x}{1-\frac{2}{3}x}\right)$$

$$y = \tan^{-1}5x - \tan^{-1}x + \tan^{-1}\frac{2}{3} + \tan^{-1}x$$

$$\boxed{\frac{dy}{dx} = \frac{5}{1+25x^2}}$$

Question

$$y = \cos^{-1}\left(\frac{\cos x + 4\sin x}{\sqrt{17}}\right)$$

$$y = \cos^{-1}\left(\frac{1}{\sqrt{17}}\cos x + \frac{4}{\sqrt{17}}\sin x\right)$$

$$\theta = \sin^{-1}\frac{4}{\sqrt{17}} = \cos^{-1}\frac{1}{\sqrt{17}}$$

$$y = \cos^{-1}(\cos\theta\cos x + \sin\theta\sin x)$$

$$= \cos^{-1}(\cos(x-\theta)) = x - \theta$$

$$\frac{dy}{dx} = 1 - \frac{d\theta}{dx} = 1 \Rightarrow \boxed{\frac{dy}{dx} = 1}$$

DIFFERENTIATION OF IMPLICIT FUNCTION

EXPLICIT FUNCTION :- If y is expressed in term of x entirely i.e. $y = f(x)$ then y is called explicit function.

eg $y = x^2+x+1$, $y = e^x$ etc.

IMPLICIT FUNCTION:—

If y depends on x and can not be expressed entirely in term of x i.e $f(x,y)=0$ then y is called implicit function of x.

eg. $y = 2x^2 = \sin(y+2x)$

$$\boxed{x^3+y^3-xy^2+3xy-1=0}$$

METHOD TO DIFFERENTIATION

1. Differentiate each term w.r.t. x.
2. Collect terms containing dy/dx.
3. Exposed $\frac{dy}{dx}$ is a function of x, y

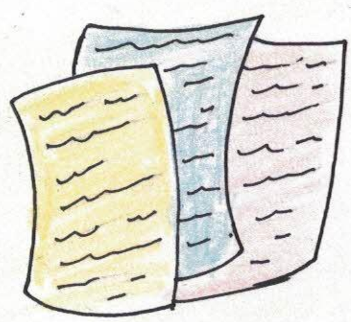
$$\frac{dy}{dx} = g(x,y)$$

Ex. $y = 2x^2 = \sin(y+2x)$

$\rightarrow$ $\frac{dy}{dx} + 4x = \cos(y+2x)\left(\frac{dy}{dx}+2\right)$

$\frac{dy}{dx}\left(1-\cos(y+2x)\right) = 2\cos(y+2x) - 4x$

$$\boxed{\frac{dy}{dx} = \frac{2\cos(y+2x)-4x}{1-\cos(y+2x)}}$$



Question

$x^{\sin y} = (\sin y)^x$

$\rightarrow$ $\sin y \log x = x \log(\sin y)$

$\log x \cos y \frac{dy}{dx} + \frac{\sin y}{x} = \frac{x}{\sin y}\frac{dy}{dx}\cos y + \log(\sin y)$

$\frac{dy}{dx}\left(\log x \cos y - \frac{x\cos y}{\sin y}\right) = \log \sin y - \frac{\sin y}{x}$

$$\boxed{\frac{dy}{dx} = \frac{\log(\sin y) - \frac{\sin y}{x}}{\log x \cos y - \frac{x \cos y}{\sin y}}}$$

Question If $\sin y = x \sin(a+y)$ then P.T.

1 $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ 2) $\frac{dy}{dx} = \frac{x^2}{1+x^2 2x\cos a}$

$\longrightarrow$ $x = \frac{\sin y}{\sin(a+y)}$

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)(1)}{\sin^2(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin(a+y-y)}{\sin^2(a+y)} \Rightarrow \boxed{\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}}$$

DIFFERENTIATION OF DETERMINANT

$$F(x) = \begin{vmatrix} f(x) & g(x) \\ u(x) & v(x) \end{vmatrix}$$

If $f(x)$, $g(x)$, $u(x)$, $v(x)$ are differentiable forms of x, then $F(x)$ is differentiable.

$$F'(x) \Rightarrow \begin{vmatrix} f'(x) & g'(x) \\ u(x) & v(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) \\ f'(x) & g'(x) \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} f'(x) & g(x) \\ u'(x) & v(x) \end{vmatrix} + \begin{vmatrix} f(x) & g'(x) \\ u(x) & v(x) \end{vmatrix}$$

Proof

$$F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\begin{vmatrix} f(x+h & g(x+h) \\ u(x+h) & v(x+h) \end{vmatrix} - \begin{vmatrix} f(x) & g(x) \\ u(x) & v(x \end{vmatrix}\right]$$

$$F'(x) = \lim_{h \to 0} \frac{1}{h}\left[\begin{vmatrix} f(x+h) & g(x+h) \\ u(x+h) & v(x+h) \end{vmatrix} - \begin{vmatrix} f(x) & g(x) \\ u(x+h) & v(x+h) \end{vmatrix}\right]$$

$$+ \begin{vmatrix} f(x) & g(x) \\ u(x+h) & v(x+h) \end{vmatrix} - \begin{vmatrix} f(x) & g(x) \\ u(x) & v(x) \end{vmatrix}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\begin{vmatrix} f(x+h) - f(x) & g(x+h) - g(x) \\ u(x+h) & v(x+h) \end{vmatrix}\right] + \frac{1}{h}\left[\begin{vmatrix} f(x) & g(x) \\ u(x+h) - u(x) & v(x+h) - v(x) \end{vmatrix}\right]$$

$$= \lim_{h \to 0}\left[\begin{vmatrix} \frac{f(x+h) - f(x)}{h} & \frac{g(x+h) - g(x)}{h} \\ u(x+h) & v(x+h) \end{vmatrix}\right] + \frac{1}{h}\left[\begin{vmatrix} f(x) & g(x) \\ u(x+h) - u(x) & v(x+h) - v(x) \end{vmatrix}\right]$$

$$\Rightarrow \lim_{h \to 0} \begin{vmatrix} f'(x) & g'(x) \\ u(x) & v(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) \\ u'(x) & v'(x) \end{vmatrix}$$

$$F(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}$$

$$F'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l'(x) & m'(x) & n'(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u'(x) & v'(x) & w'(x) \end{vmatrix}$$

Question

If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 1 & 2 & -1 \\ k & k^2 & k+1 \end{vmatrix}$, k = constant

Find $\frac{d^3}{dx^3}(f(x))\Big|_{at\ x=0}$

$\rightarrow \frac{d}{dx} f(x) = \begin{vmatrix} 3x^3 & \cos x & -\sin x \\ 1 & 2 & -1 \\ k & k^2 & k+1 \end{vmatrix}$

$\frac{d}{dx^2} f(x) = \begin{vmatrix} 6x & -\sin x & -\cos x \\ 1 & 2 & -1 \\ k & k^2 & k+1 \end{vmatrix}$

$\frac{d^3}{dx^3} f(x) = \begin{vmatrix} 6 & -\cos x & \sin x \\ 1 & 2 & -1 \\ k & k^2 & k+1 \end{vmatrix}$

$= \begin{vmatrix} 6 & -1 & 0 \\ 1 & 2 & -1 \\ k & k^2 & k+1 \end{vmatrix}$ $\Rightarrow 6(2k+2+k^2)+1(k+1+k)$

$\Rightarrow 12k+12+6k^2+2k+1$

$= \boxed{6k^2+16k+13}$

NOTES

Question

if $f(x) = \begin{vmatrix} x^n & \sin x & -\cos x \\ n! & \sin(\frac{n\pi}{2}) & \cos(\frac{n\pi}{2}) \\ 2014 & (2014)^2 & (2014)^3 \end{vmatrix}$ n = constant $= (2m+1)$

$\frac{d^n}{dx^n} f(x)\Big|_{at\ x=0}$

$\rightarrow$

$f(x) \begin{vmatrix} x^n & \sin x & -\cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ 2014 & (2014)^2 & (2014)^3 \end{vmatrix}$

$$\frac{d f(x)}{dx} = \begin{vmatrix} nx^{n-1} & \cos x & \sin x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ 2014 & (2014)^2 & (2014)^3 \end{vmatrix}$$

$\cos x \to 1$
$-\sin x \to 2$
$-\cos x \to 3$
$\sin x \to 4$
$\cos x \to 5$

$$\frac{d^n f(x)}{dx^n} = \begin{vmatrix} n(n-1)\ldots(n-n+1) & \pm\cos x & \pm\sin x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ 2014 & (2014)^2 & (2014)^3 \end{vmatrix}$$

$\sin x \to 1$
$\cos x \to 2$
$-\sin x \to 3$
$-\cos x \to 4$
$\sin x \to 5$

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x=0} = \begin{vmatrix} n! & \pm 1 & 0 \\ n! & \pm 1 & 0 \\ (2014) & (2014)^2 & (2014)^3 \end{vmatrix} \Rightarrow \begin{vmatrix} n! & \pm 1 & 0 \\ n! & \pm 1 & 0 \\ (2014) & (2014)^2 & (2014)^3 \end{vmatrix}$$

$$\boxed{= 0}$$

DIFFERENTIATION OF INVERSE FUNCTION

$y = f(x) \qquad x = g(y)$

$$1 = \frac{\Delta y}{\Delta x} \times \frac{\Delta x}{\Delta y}$$

$$\lim_{\Delta x \to 0} \Rightarrow \frac{dy}{dx} \times \frac{dx}{dy} = 1 \Rightarrow \left.\frac{dx}{dy}\right|_{x=a} = \left.\frac{1}{dy/dx}\right|_{x=b}$$

$x=a \downarrow$ Input

$x=b \downarrow$ Output

Question

$y = x^3 + x^5$

Let $g(x)$ be the inverse of y, find $g'(x)$ at $x = 2$.

$y = x^3 + x^5 \qquad x^3 + x^5 = 2$

$$\frac{dy}{dx} = 3x^2+5x^4 \qquad x=1$$

$$g'(x) = \frac{1}{3x^2+5x^4}\Bigg|_{x=1} \boxed{\Rightarrow \frac{1}{8}}$$

Question If $f(x) = e^{x^3+x^2+x}$, and $g(x)$ be inverse of $f(x)$ then find $g'(x)$ at $x = e^3$.

$\rightarrow$ $f'(x) = e^{x^3+x^2+x} \times (3x^2+2x+1)$

$$e^{x^3+x^2+x} = e^3 \Rightarrow x = \underline{1}$$

$$g'(x) = \frac{1}{f'(x)}\Bigg|_{x=1} \Rightarrow \frac{1}{e^{x^3+x^2+x} \times (3x^2+2x+1)}\Bigg|_{x=1}$$

$$= \frac{1}{e^3 \times 6} = \boxed{\frac{1}{6e^3}}$$

Question If $f(x) = e^x + x^3 - 1$, and $g(x)$ is inverse of $f(x)$ find $g'(e)$.

$\rightarrow$ $e^x + x^3 - 1 = e \Rightarrow x = 1$

$$g'(x)\Big|_{x=e} = \frac{1}{f'(x)}\Bigg|_{x=1} = \frac{1}{e^x+3x^2}\Bigg|_{x=1} = \boxed{\frac{1}{e+3}}$$

SECOND DERIVATIVE OF INVERSE FUNCTION

If $f(x)$ & $g(y)$ are inverse of each other,

$$g'(y) = \frac{1}{f'(x)}$$

$$g''(y) = \frac{d}{dy}\left(\frac{1}{f'(x)}\right) = \frac{d}{dx}\left(\frac{1}{f'(x)}\right) \times \frac{dx}{dy} \Rightarrow \frac{-f''(x)}{[f'(x)]^2} \times \frac{1}{\frac{dy}{dx}}$$

$$g''(y) = \frac{-f''(x)}{[f'(x)]^3}$$

Question

Find 3rd derivative of inverse of f(x).

$$g''(y) = \frac{-f''(x)}{[f'(x)]^3}$$

$$g'''(y) = -\frac{d}{dy}\left(\frac{f''(x)}{[f'(x)]^3}\right) = -\frac{d}{dx}\left(\frac{f''(x)}{[f'(x)]^3}\right) \times \frac{dx}{dy}$$

$$= -\frac{[f'(x)]\, f'''(x) + 3f''(x)^2}{[f'(x)]^5}$$

$$\boxed{g'''(y) = \frac{3[f''(x)]^2 - f'''(x) f'(x)}{[f'(x)]^5}}$$

Question

If $f(x) = e^x + x^3 - x$ and $g(y)$ be inverse of $f(x)$. then find $g''(e)$.

$\rightarrow$

$$g'(y) = \frac{1}{f'(x)} = \frac{1}{e^x + 3x^2 - 1}$$

$$g''(y) = \frac{-(e^x + 6x)}{(e^x+3x^2-1)^2} \times \frac{g''(x)|_{x=e}}{(e^x+3x^2-1)}$$

$e^x + x^3 - x = e$

$x = 1$

$$= \frac{-(e+6)}{(e+3-1)^3} = \boxed{\frac{-e+6}{(e+2)^3}}$$

Question

If $f(x) = 1 + x^3$, $g(y)$ be inverse of $f(x)$ then find $g'''(2)$.

$\rightarrow$

$$g'(x) = \frac{1}{3x^2}$$

$$g''(x) = \frac{-6x}{(3x^2)^2} \times \frac{1}{(3x^2)} = \frac{-6x}{(3x^2)^3} = \frac{-6}{3^3 x^5}$$

$$g'''(x) = \frac{6 \times 3^3 \times 5x^4}{(3)^6 \times x^{10}} \times \frac{1}{3x^2} = \frac{36}{27x^8 \times 3x^2}$$

$$\boxed{= \frac{10}{27}}$$

Always just multiply by $g'(x)$ in $g''(x)$ and $g'''(x)$.

Ex:- $\sin\theta + 3\sin 3\theta + 5\sin 5\theta + \ldots\ldots + (2K+1)\sin(2K+1)\theta$

$\longrightarrow -[\cos\theta + \cos 3\theta + \ldots\ldots \cos(2K+1)\theta]$

$$= -\frac{\sin(K+1)\theta}{\sin\theta} \cos\left(\frac{\theta + 2K\theta + \theta}{2}\right)$$

$$= -\frac{2\sin(K+1)\theta}{2\sin\theta} \cos(K+1)\theta = \boxed{-\frac{\sin 2(K+1)\theta}{2\sin\theta}}$$

Ex:- $\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \ldots\ldots + \frac{2^n x^{2^n - 1}}{1+x^{2^n}}$

$\longrightarrow$ This is derivative of :-

$\ln(1+x) + \ln(1+x^2) + \ln(1+x^4) + \ldots\ldots \ln(1+x^{2^n})$

$\Rightarrow \ln[(1+x)(1+x^2)(1+x^4)\ldots\ldots(1+x^{2^n})]$

$$= (1+x)(1+x^2)(1+x^4)\ldots\ldots(1+x^{2^n}) \times \frac{(1-x)}{(1-x)}$$

$$= \frac{(1-x^{2^n})(1+x^{2^n})}{(1-x)} = \frac{(1-x^{2^n})^2}{1-x} = \frac{(1-x)^{2^{n+1}}}{1-x}$$

$$\ln\left(\frac{1-x}{1-x}\right)^{2^{n+1}}$$

$$= \ln((1-x)^{2^{n+1}}) - \ln(1-x)$$

$$= \frac{1}{1-x^{2^{n+1}}} \times -(2^{n+1}x^{2^{n+1}-1}) - \frac{1}{1-x} \times -1$$

$$\Rightarrow \boxed{\frac{-2^{n+1}x^{2^{n+1}-1}}{1-x^{2^{n+1}}}+\frac{1}{1-x}}$$

If $0<x<1$ then find.

$$\frac{1-2x}{1-x+x^2}+\frac{2x-4x^3}{1-x^2+x^4}+\frac{4x^3-8x^7}{1-x^4+x^8}+\dots\dots\infty$$

$$\rightarrow -\ln(1-x+x^2)-\ln(1-x^2+x^4)-\ln(1-x^4+x^8)+\dots\dots\infty$$

Multiply & devide by x^2+x+1

$$-\left[\ln(1-x+x^2)(1-x^2+x^4)\dots\infty\right]$$

$$=\frac{-(1-x+x^2)(1-x^2+x^4)(1-x^4+x^8)\dots(1+x+x^2)}{(1+x+x^2)}$$

$$=\frac{-(1+x^2+x^4)(1-x^2+x^4)\dots\infty}{(1+x+x^2)}$$

If no. of term are n.

$$-(1+x^{2^{n+1}}+x^{2^{n+2}}) \qquad x\in(0,1),\ n\to\infty$$

$$=-\ln\left(\frac{-1}{1+x+x^2}\right)$$

$$=+\frac{1+2x}{(1+x+x^2)} \Rightarrow \boxed{\frac{1+2x}{1+x+x^2}}$$

Question Find the coefficient of x in :-

$$f(x) = \begin{vmatrix} (1+x)^{a_1b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_3} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \\ (1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3} \end{vmatrix}$$

$$\rightarrow \quad f'(x) = \begin{vmatrix} a_1b_1(1+x)^{a_1b_1-1} & a_2b_2(1+x)^{a_1b_2-1} & (a_1b_3)(1+x)^{a_1b_3-1} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \\ (1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3} \end{vmatrix}$$

At $x=0$, $+$ 0 $+$ 0

$\boxed{\rightarrow \quad 0}$

SINX

DIFFERENTIATION OF ODD / EVEN FUNCTION

1. If $f(x)$ is an odd function, then its derivative of $f'(x)$ is an even function within its domain.

$$f(-x) = -f(x)$$

$$f'(-x) \times -1 = -f'(x)$$

$$-f'(-x) = -f'(x)$$

$$\boxed{f'(-x) = f'(x)}$$

2. If $f(x)$ is an even function, then its derivative is odd within its domain.

$$f(x) = +f(-x)$$
$$f'(x) = f'(-x) \times -1$$
$$f'(x) = -f'(-x) \Rightarrow \boxed{f'(-x) = -f'(x)}$$

Note

If $f'(x)$ is an odd function then $f(x)$ must be even functional. But if $f'(x)$ is even function then we cannot say about $f(x)$.

Question

$$f(\theta) = \cos\theta + \cos 2\theta + \cos 3\theta + \dots\dots \cos n\theta$$
$$f(x) = \frac{1}{1+x} + \frac{2}{1+x^2} + \frac{4}{1+x^4} + \dots\dots \frac{2^n}{1+x^{2^n}}$$

then P. T. $\tan\theta + 2\tan 2\theta + 3\tan 3\theta + \dots\dots n\tan n\theta$

$$= -\frac{f'(\theta)}{f(\theta)}$$

ALGEBRA OF DIFFERENTIABLE FUNCTIONS

If $f(x)$ & $g(x)$ be two differentiable function at at $x = a$ then-

1. $c\,f(x) \longrightarrow$ differentiable at $x = a$.
2. $f(x) \pm g(x) \longrightarrow$ diff. at $x = a$
3. $f(x)\,g(x) \longrightarrow$ diff. at $x = a$
4. $\dfrac{f(x)}{g(x)} \longrightarrow$ (diff. at $x = a$) $(g'(x) \neq 0)$

Sum of finite no. of diff^n function at a point $x=a$ is also differentiable at that pt.

Product of finite no. of differentiable function at a point $x=a$ is also differentiable at that Point.

* If $f(x)$ is diff.^{n} but $g(x)$ is non diff^n at $x>a$ then –

1(i) $f(x) \pm g(x) \longrightarrow$ Non diff^n at $x=a$

2(ii) $f(x)\, g(x) \longrightarrow$ No Comment at $x=a$

Ex. $f(x) = x^3 \qquad g(x) = \text{sgn}(x)$

$f(x)$ differentiable at $x=0$, $g(x)$ non diff^n at $x=0$

$$f(x)\,g(x) = \begin{cases} x^3, & x>0 \\ 0, & x=0 \\ -x^3, & x<0 \end{cases}$$

Hence $f(x)\,g(x)$ is diff^n at $x=0$

iii $\dfrac{f(x)}{g(x)} \longrightarrow$ No comment at $x=a$

3(i) If $f(x)$ & $g(x)$ both are non differentiable, Then either $(f(x)+g(x)$ or $f(x)-g(x)$ may be differentiable but both cannot be simultaneous differentiable.

Proof

$f(x) + g(x) \rightarrow$ diff

$f(x) - g(x) \rightarrow$ diff

$2f(x) \rightarrow$ diff (wrong)

H.P

EX.

$f(x) = [x] \quad g(x) = [x]$ non diff.^{n} at $x=0$,

$f(x) + g(x) = x \rightarrow$ diff^n at $x=0$

$f(x) - g(x) = 2[x] - x \rightarrow$ non diff^n at $x=0$

(ii) $f(x)\,g(x) \longrightarrow$ No comment at $x=a$

(iii) $\dfrac{f(x)}{g(x)} \longrightarrow$ No comment

4. If $f(x)$ is diff.^{n} at $x=a$ and $g(x)$ is diff^n at $f(a)$, then $gof(x)$ is diff^n at $x=a$.

5. Composition of finite no. of diff^n function at a point is also a diff^n function.

6. If $f(x)$ is continuous and diff^n in $[a,b]$ and range of $f(x)$ is $[A,B]$ $g(x)$ is also cont. & diff^n in $[A,B]$ $g\,of(x)$ is also continuous and diff^n in $[A,B]$.

7. if two function $f(x)$ & $g(x)$ are differentiabe every-where then their composition is also diff^n everywhere.

Question

P.T. $f(x) = \dfrac{\sin x}{1+x^2}\left|\dfrac{\sin x}{1+x}\right|$ is diff^n $\forall x$.

$f(x) = \dfrac{\sin x}{1+x^2}$ $\quad g(x) = x|x| \rightarrow$ cont^n & diff. all x.

cont. & diff^n for all x

$g\,of(x) = h(x) \rightarrow$ diff^n & cont^n for all x. H.P.